NETWORK AND SYSTEM

UNIT-1

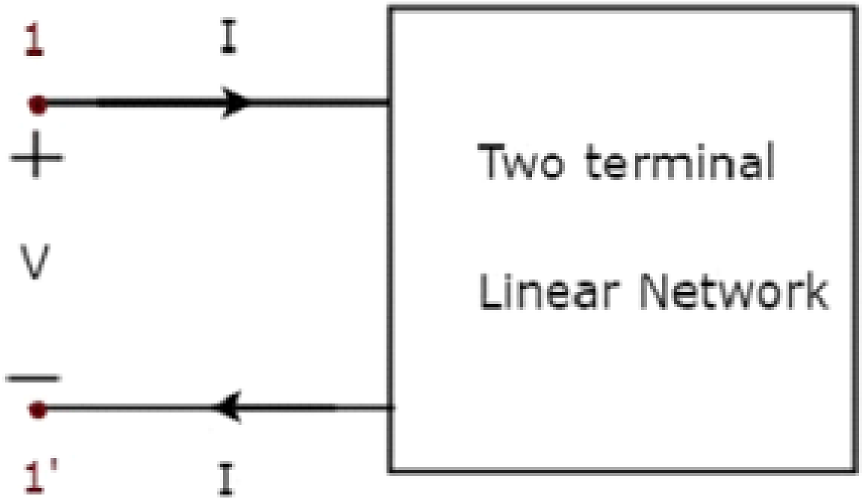
Network

An **electrical network** is an interconnection of **electrical** components (e.g., batteries, resistors, inductors, capacitors, switches, transistors).

Network analysis is the process of finding the voltages across, and the currents through, all network components.

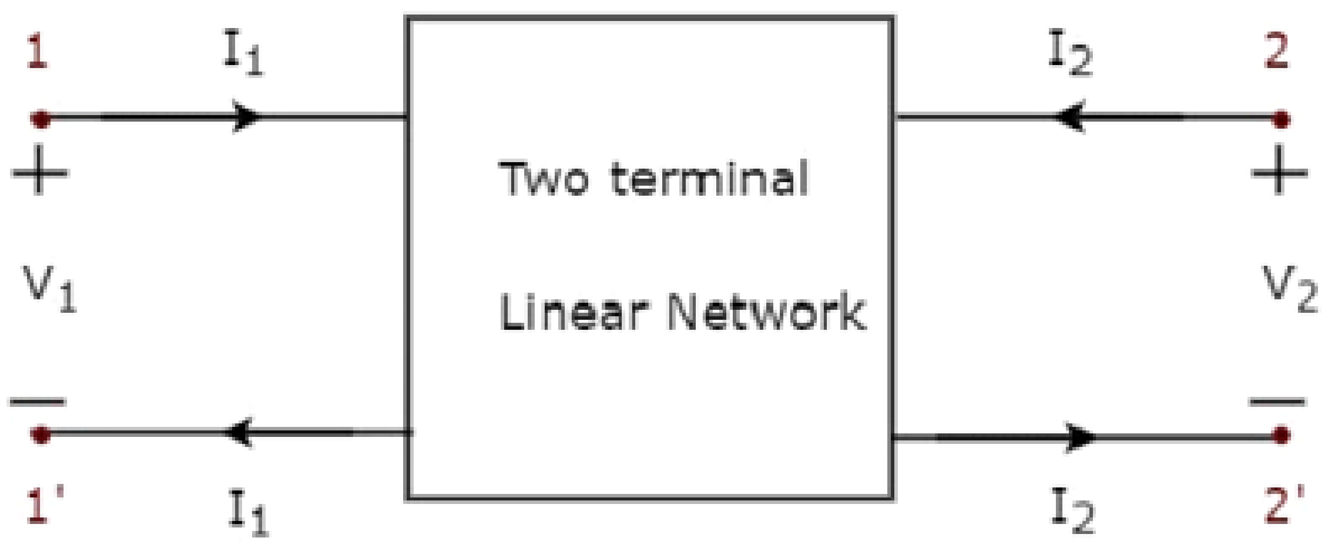
In general, it is easy to analyze any electrical network, if it is represented with an equivalent model, which gives the relation between input and output variables. For this, we can use **two port network** representations. As the name suggests, two port networks contain two ports. Among which, one port is used as an input port and the other port is used as an output port. The first and second ports are called as port1 and port2 respectively.

One port network is a two terminal electrical network in which, current enters through one terminal and leaves through another terminal. Resistors, inductors and capacitors are the examples of one port network because each one has two terminals. One port network representation is shown in the following figure.



Here, the pair of terminals, 1 & 1’ represents a port. In this case, we are having only one port since it is a one port network.

Similarly, **two port network** is a pair of two terminal electrical network in which, current enters through one terminal and leaves through another terminal of each port. Two port network representation is shown in the following figure.



Here, one pair of terminals, 1 & 1' represents one port, which is called as **port1** and the o

ther pair of terminals, 2 & 2' represents another port, which is called as **port2**.

There are **four variables** V_1, V_2, I_1 and I_2 in a two port network as shown in the figure. Out of which, we can choose two variables as independent and another two variables as dependent.

Types of network

1. Symmetrical network
2. Asymmetrical network
3. Balanced network
4. Unbalanced network
5. T network
6. Pi network
7. Lattice network
8. L-network
9. Bridge T-network

1.Symmetrical network- A symmetrical network is one whose electrical characterstics do not change when its input And output terminals are interchanged.

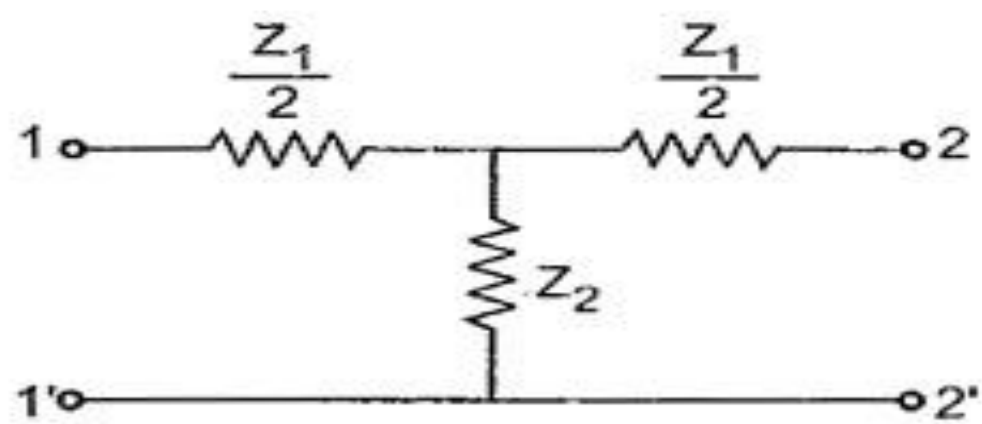


Fig. 8.7 Symmetrical T network

Also they offered same impedance at input and output terminals.

2.Asymmetrical network- A asymmetrical network is one whose electrical characterstics changes when its input And output terminals are interchanged.

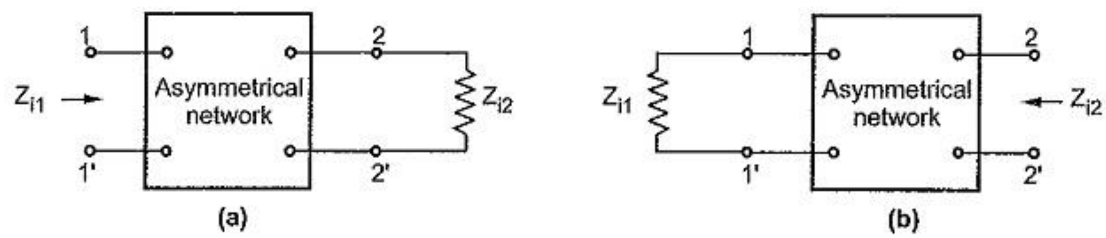
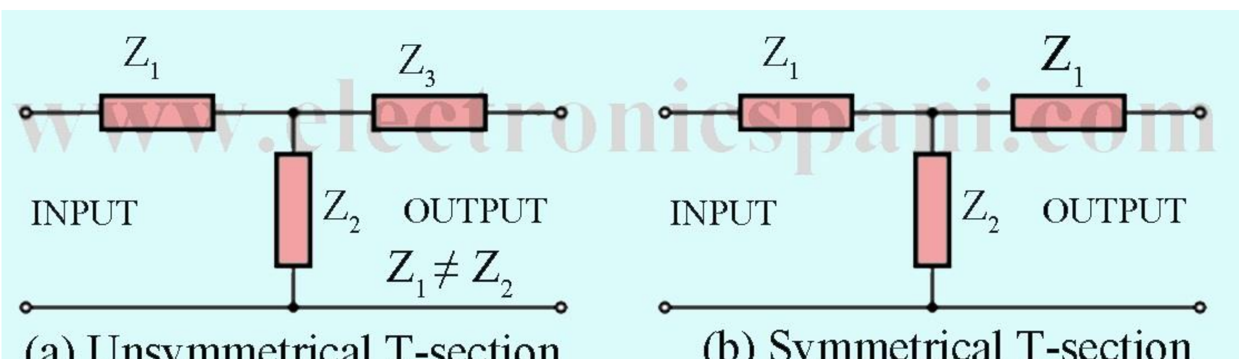


Fig. 8.5 Image impedances of asymmetrical network

A step up transformer is good example of asymmetrical network.on interchanging of both terminals characterstics of transformer also changed.

5. T network-

When a electrical network section looks like a *"T"*, it is known as *T-Section* Figure 1 (a) and (b) represent *unsymmetrical and symmetrical* T-sections.

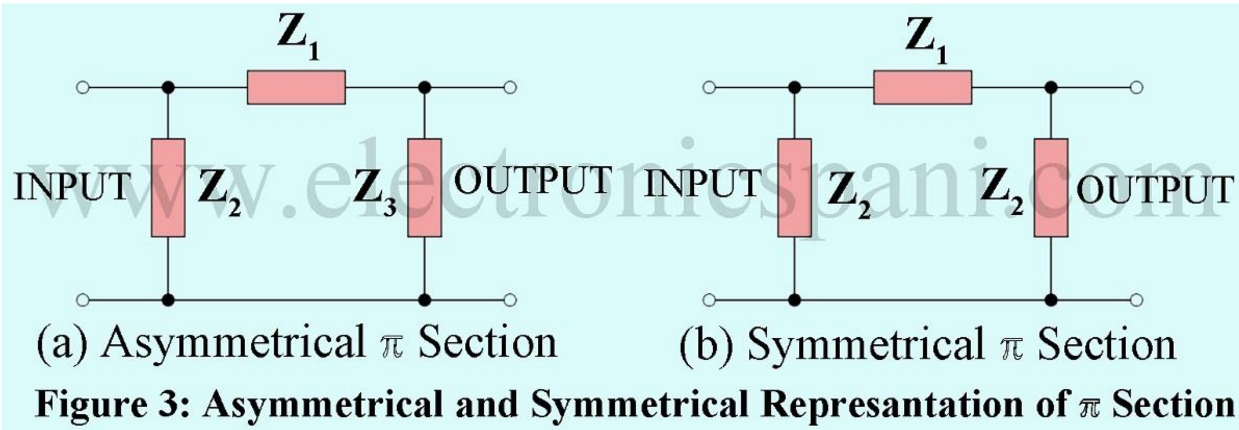


(a) Unsymmetrical T-section (b) Symmetrical T-section

Figure 1: Symmetrical and Unsymmetrical T-Section

2. π-Section

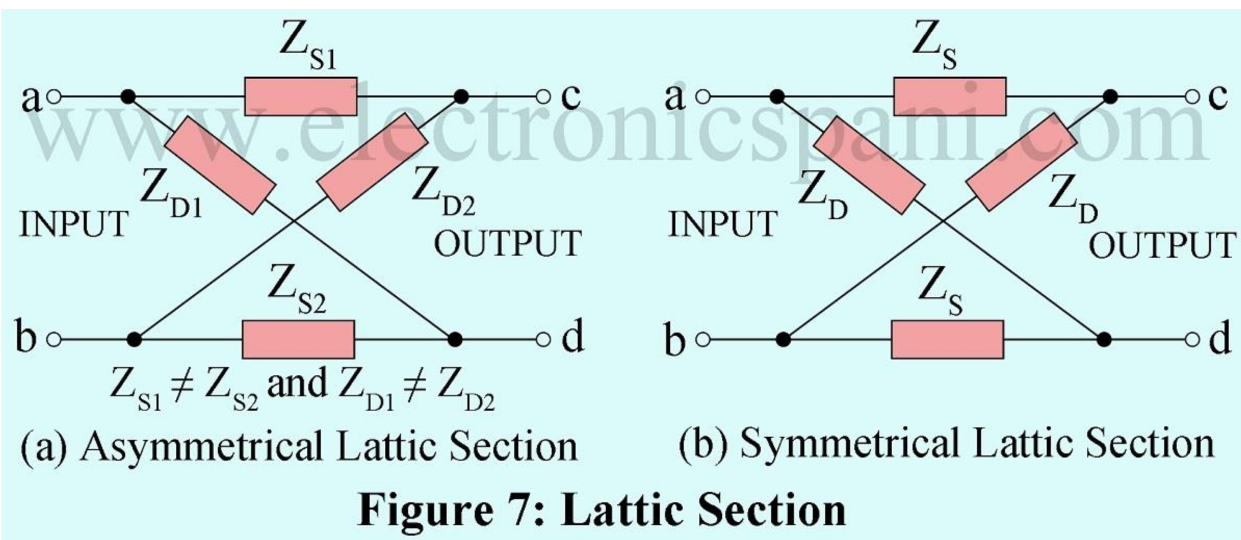
In an identical manner, symmetrical and asymmetrical π section (i.e., the section whose structure looks like a π) can be configured (Figure 3). In asymmetrical π section, the shunt arm impedances are not identical while for the symmetrical π section, the shunt arm impedances must be identical.



5. Lattice Section

It is only network which is already in balanced form.it has two arms i.e.series and shunt arm.

Figure 7 represents a lattice section where the series arms are given by the impedances between (a and c) and (b and d). The diagonal arms are frequently termed as cross arms. Figure 7(a) represents are asymmetrical lattice section while figure 7(b) represents symmetrical lattice section. The lattice section is usually a balanced structure.



9. L-section

When the network section looks like “L”, the configuration is termed as L section. It may be observed that L-section is merely a specific case of the asymmetrical T-section with one series arm equal to zero value or of the asymmetrical π section with one shunt arm equal to infinity,the resultant network is called L-section.

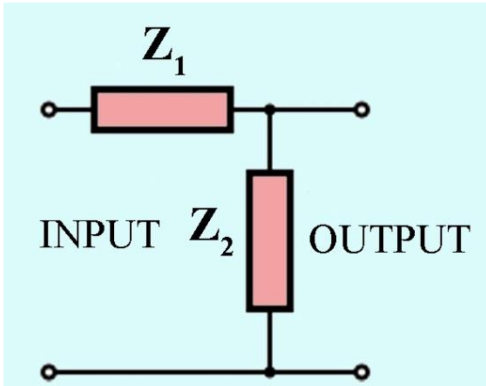


Figure 5: Representation of an L-section

10. Bridged T-network-

When the series arms of a T-section are bridged by an impedance, the section is termed as bridges T-section Figure represent symmetrical bridge T-section.

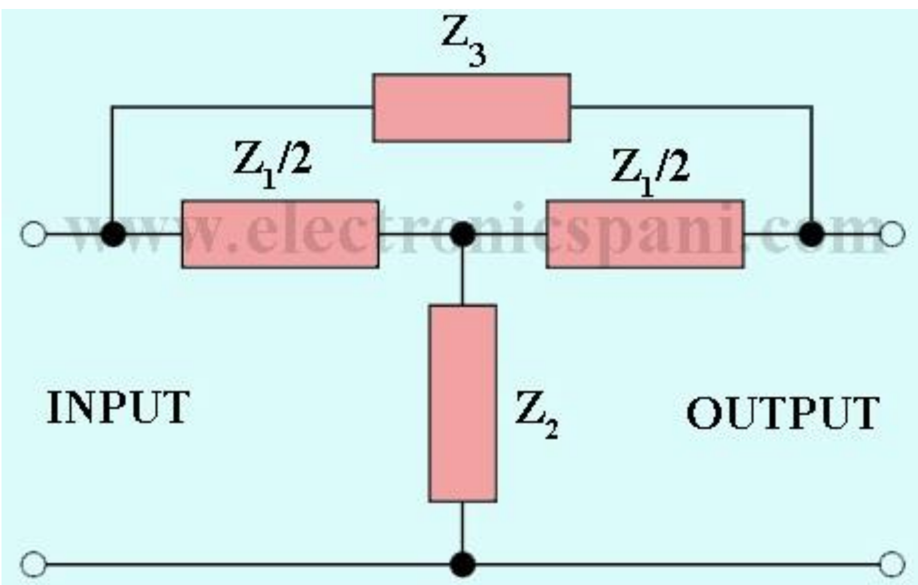


Figure 8 Bridge T-representation

Symmetrical pi Network in Network Analysis:

The Symmetrical pi Network in Network Analysis is another important network in line transmission fulfilling the conditions of total series and shunt arm impedances as Z_1 and Z_2 respectively. Thus the series arm impedance of a it network is selected as Z_1 and to have a total shunt arm impedance of Z_2, each shunt arm impedance is selected as $2Z_2$ as shown in the Fig. 8.15.

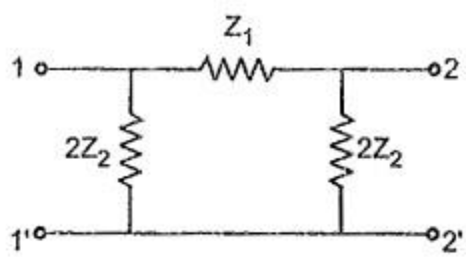


Fig. 8.15 Symmetrical π network

Similar to the symmetrical T network, let us derive the expressions for the characteristic impedance (Z_0) and propagation constant (γ) of the Symmetrical pi Network in Network Analysis.

Characteristic Impedance (Z_0):

(A) In terms of series and shunt arm impedances

Consider a symmetrical π network terminated at its output terminals with its characteristic impedance Z_0 as shown in the Fig. 8.16.

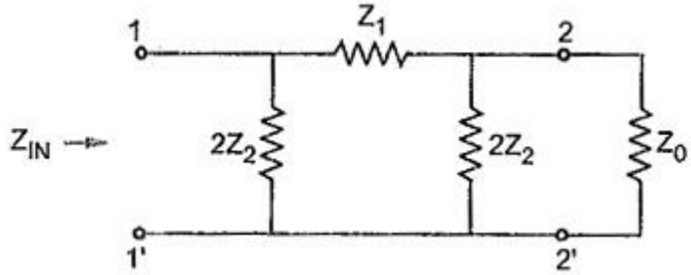


Fig. 8.16 A symmetrical π network terminated with Z_0

By the property of the symmetrical network, the input impedance of such network terminated with Z_0 at other port is equal to Z_0. The input impedance of a symmetrical π network is given by

$$Z_{IN} = Z_0 = 2Z_2 \parallel [Z_1 + (2Z_2 \parallel Z_0)]$$

$$Z_0 = 2Z_2 \parallel \left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]$$

$$Z_0 = 2Z_2 \parallel \left[\frac{Z_1(2Z_2 + Z_0) + 2Z_2 Z_0}{2Z_2 + Z_0}\right]$$

$$Z_0 = \frac{2Z_2\left[\dfrac{2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}{2Z_2 + Z_0}\right]}{2Z_2 + \dfrac{2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}{2Z_2 + Z_0}}$$

$$Z_0 = \frac{2Z_2(2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0)}{2Z_2(2Z_2 + Z_0) + 2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}$$

$$4Z_2^2 Z_0 + 2Z_2 Z_0^2 + 2Z_1 Z_2 Z_0 + Z_1 Z_0^2 + 2Z_2 Z_0^2 = 4Z_1 Z_2^2 + 2Z_1 Z_2 Z_0 + 4Z_2^2 Z_0$$

$$4Z_2 Z_0^2 + Z_1 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2 (Z_1 + 4Z_2) = 4Z_1 Z_2^2$$

$$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Multiplying numerator and denominator by the factor $Z_1/4$,

$$Z_0^2 = \frac{Z_1^2 Z_2^2}{\dfrac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_0^2 = \frac{Z_1^2 Z_2^2}{Z_{0_T}^2} \qquad \ldots (1)$$

Here the characteristic impedance of it section is indicated by $Z_{0\pi}$ and that of T section by Z_{0T}.
Taking square root on both the sides,

$$Z_{0_\pi} = \frac{Z_1 Z_2}{Z_{0T}} \quad \dots (2)$$

(B) In terms of open and short circuit impedances

Consider Symmetrical pi Network in Network Analysis shown in the Fig. 8.17 (a) and Fig. 8.17 (b).

Consider Fig. 8.17 (a),

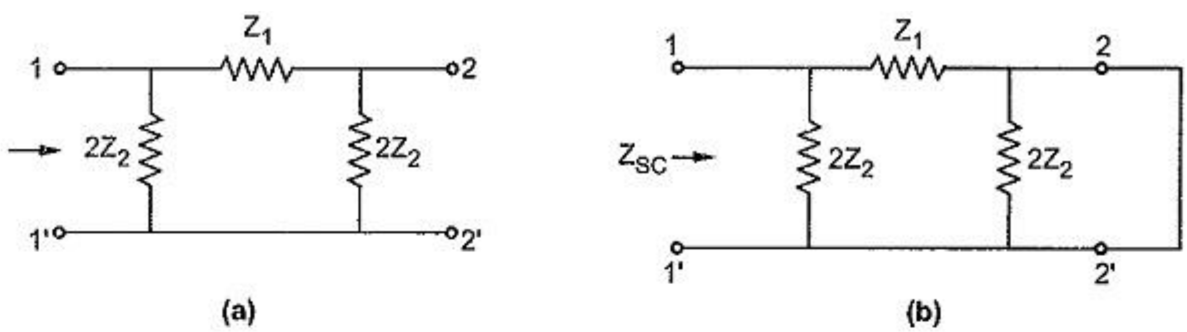


Fig. 8.17 Open and short circuit impedances of symmetrical π network

$$Z_{OC} = 2Z_2 \parallel (Z_1 + 2Z_2) = \frac{2Z_2(Z_1 + 2Z_2)}{Z_1 + 4\,Z_2} \quad \dots (3)$$

Consider Fig. 8.17 (b),

$$Z_{SC} = 2Z_2 \parallel Z_1 = \frac{2Z_1 Z_2}{(Z_1 + 2Z_2)} \qquad \dots (4)$$

Multiplying equations (3) and (4), we can write,

$$Z_{OC} \cdot Z_{SC} = \frac{2Z_2(Z_1 + 2Z_2)}{(Z_1 + 4Z_2)} \cdot \frac{2Z_1 Z_2}{(Z_1 + 2Z_2)}$$

$$Z_{OC} \cdot Z_{SC} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

$$Z_{OC} \cdot Z_{SC} = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2} \cdot \frac{\left(\frac{Z_1}{4}\right)}{\left(\frac{Z_1}{4}\right)} = \frac{Z_1^2\, Z_2^2}{\frac{Z_1^2}{4} + Z_1 Z_2} = Z_0^2$$

Thus the characteristic impedance of symmetrical π network is given by

$$Z_0 = Z_{0_\pi} = \sqrt{Z_{OC} \cdot Z_{SC}} \qquad \dots (5)$$

Characteristic Impedance

Symmetrical T Network in Network Analysis:

In line transmission theory, the symmetrical T network is the most frequently used network. The condition in the symmetrical T network is that the total series arm impedance and shunt arm impedance must be Z_1 and Z_2 respectively. To have a total series arm impedance of $Z_{1'}$ the two series arm impedances must be selected as $Z_1/2$ each as shown in the Fig. 8.7.

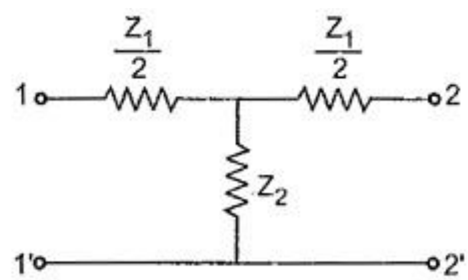


Fig. 8.7 Symmetrical T network

Let us derive the expressions for the characteristic impedance (Z_0) and propagation constant (γ) in terms of the network elements.

Characteristic Impedance (Z_0):

(A) In terms of series and shunt arm impedances

Consider a symmetrical T network terminated at its output terminal with its characteristic impedance as shown in the Fig. 8.8.

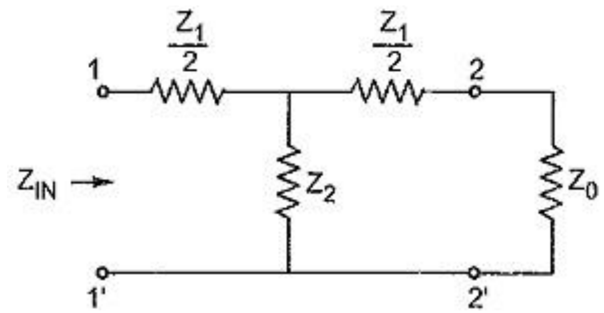


Fig. 8.8 Symmetrical T network terminated in Z_0

By the property of the symmetrical network, the input impedance of such network terminated in Z_0 at other port is equal to Z_0.
The input impedance of a T network is given by,

$$\therefore \quad Z_{1N} = Z_0 = \frac{Z_1}{2} + \left[Z_2 \parallel \left(\frac{Z_1}{2} + Z_0\right)\right]$$

$$\therefore \quad Z_0 = \frac{Z_1}{2} + \frac{Z_2\left(\frac{Z_1}{2} + Z_0\right)}{Z_2 + \frac{Z_1}{2} + Z_0}$$

$$\therefore \quad Z_0\left(Z_2 + \frac{Z_1}{2} + Z_0\right) = \frac{Z_1}{2}\left(Z_2 + \frac{Z_1}{2} + Z_0\right) + \frac{Z_1 Z_2}{2} + Z_2 Z_0$$

$$\therefore \quad Z_2 Z_0 + \frac{Z_1 Z_0}{2} + Z_0^2 = \frac{Z_1 Z_2}{2} + \frac{Z_1^2}{4} + \frac{Z_1 Z_0}{2} + \frac{Z_1 Z_2}{2} + Z_2 Z_0$$

$$\therefore \quad Z_0^2 = \frac{Z_1^2}{4} + Z_1 Z_2 \qquad \dots (1)$$

$$\therefore \quad Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} \qquad \dots (2)$$

CHARACTERSTICS IMPEDANCE

Symmetrical pi Network analysis

The Symmetrical pi Network in Network Analysis is another important network in line transmission fulfilling the conditions of total series and shunt arm impedances as Z_1 and Z_2 respectively. Thus the series arm impedance of a it network is selected as Z_1 and to have a total shunt arm impedance of Z_2, each shunt arm impedance is selected as $2Z_2$ as shown in the Fig. 8.15.

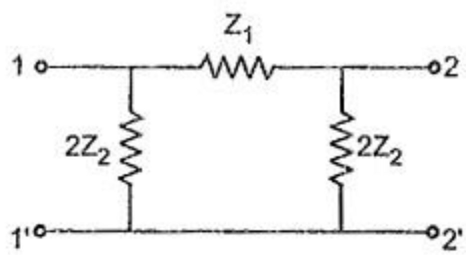


Fig. 8.15 Symmetrical π network

Similar to the symmetrical T network, let us derive the expressions for the characteristic impedance (Z_0) and propagation constant (γ) of the Symmetrical pi Network in Network Analysis.

Characteristic Impedance (Z_0):

(A) In terms of series and shunt arm impedances

Consider a symmetrical π network terminated at its output terminals with its characteristic impedance Z_0 as shown in the Fig. 8.16.

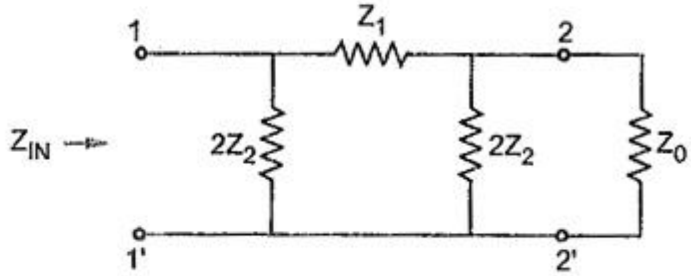


Fig. 8.16 A symmetrical π network terminated with Z_0

By the property of the symmetrical network, the input impedance of such network terminated with Z_0 at other port is equal to Z_0.

The input impedance of a symmetrical π network is given by

$$Z_{IN} = Z_0 = 2Z_2 \parallel [Z_1 + (2Z_2 \parallel Z_0)]$$

$$Z_0 = 2Z_2 \parallel \left[Z_1 + \frac{2Z_2 Z_0}{2Z_2 + Z_0}\right]$$

$$Z_0 = 2Z_2 \parallel \left[\frac{Z_1(2Z_2 + Z_0) + 2Z_2 Z_0}{2Z_2 + Z_0}\right]$$

$$Z_0 = \frac{2Z_2\left[\dfrac{2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}{2Z_2 + Z_0}\right]}{2Z_2 + \dfrac{2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}{2Z_2 + Z_0}}$$

$$Z_0 = \frac{2Z_2(2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0)}{2Z_2(2Z_2 + Z_0) + 2Z_1 Z_2 + Z_1 Z_0 + 2Z_2 Z_0}$$

$$4Z_2^2 Z_0 + 2Z_2 Z_0^2 + 2Z_1 Z_2 Z_0 + Z_1 Z_0^2 + 2Z_2 Z_0^2 = 4Z_1 Z_2^2 + 2Z_1 Z_2 Z_0 + 4Z_2^2 Z_0$$

$$4Z_2 Z_0^2 + Z_1 Z_0^2 = 4Z_1 Z_2^2$$

$$Z_0^2 (Z_1 + 4Z_2) = 4Z_1 Z_2^2$$

$$Z_0^2 = \frac{4Z_1 Z_2^2}{Z_1 + 4Z_2}$$

Multiplying numerator and denominator by the factor $Z_1/4$,

$$Z_0^2 = \frac{Z_1^2 Z_2^2}{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_0^2 = \frac{Z_1^2 Z_2^2}{Z_{0_T}^2} \qquad \dots (1)$$

Here the characteristic impedance of it section is indicated by $Z_{0\pi}$ and that of T section by Z_{0T}.

Taking square root on both the sides,

$$Z_{0_\pi} = \frac{Z_1 Z_2}{Z_{0T}} \qquad \dots (2)$$

Propagation Constant (γ):

Consider correctly terminated symmetrical π network as shown in the Fig. 8.19.

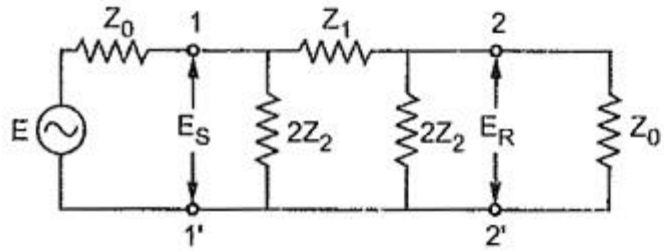


Fig. 8.19 Correctly terminated symmetrical π network

As the network is symmetrical, by definition,

$$\frac{I_S}{I_R} = \frac{E_S}{E_R} = e^{\gamma}$$

By potential divider rule,

$$E_R = E_S\left[\frac{(2Z_2 || Z_0)}{Z_1 + (2Z_2 || Z_0)}\right]$$

$$E_R = E_S\left[\frac{\dfrac{2Z_2Z_0}{2Z_2 + Z_0}}{Z_1 + \dfrac{2Z_2Z_0}{2Z_2 + Z_0}}\right]$$

$$E_R = E_S\left[\frac{2Z_2Z_0}{Z_1\ (2Z_2 + Z_0) + 2Z_2Z_0}\right]$$

$$\frac{E_S}{E_R} = e^{\gamma} = \left[\frac{2Z_2Z_0 + 2Z_1Z_2 + Z_1Z_0}{2Z_2Z_0}\right]$$

$$e^{\gamma} = 1 + \frac{Z_1}{Z_0} + \frac{Z_1}{2Z_2}$$

Rearranging the terms,

$$e^{\gamma\pi} = 1 + \frac{Z_1}{2Z_2} + \frac{Z_1}{Z_{0\pi}} \qquad \dots (1)$$

Asymmetrical Network in Network Analysis:

An asymmetrical network has following electrical properties,

- **Iterative impedance**
- **Image impedance**
- **Image transfer constant**

Iterative impedance (Z'_0):

Consider that infinite asymmetrical networks having identical electrical properties are connected in cascade as shown in the Fig. 8.4 (a) and (b).

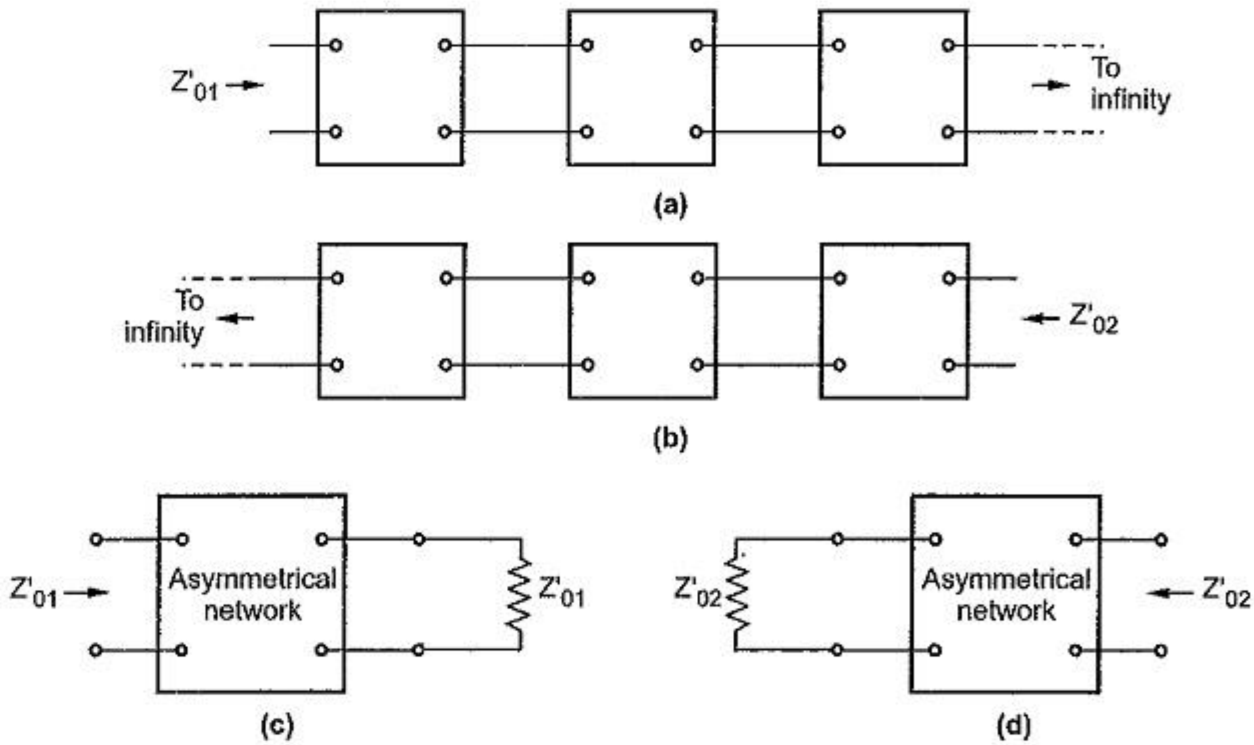


Fig. 8.4 Iterative impedances of asymmetrical network

The iterative impedance is the impedance measured at one pair of terminals of the network in the chain of infinite networks as shown in the Fig. 8.4 (a) and (b). This is the impedance measured at any pair of terminals of the network when other pair of terminals is terminated in the impedance of the same value as shown in the Fig. 8.4 (c) and (d).

The iterative impedances for any asymmetrical network are of different values when measured at different ports of the network. The iterative impedances are represented by Z'_{01} and Z'_{02} respectively at port 1 and port 2.

Image impedances (Z_i):

Similar to the iterative impedances, the image impedances are also of different values at different ports. Let the image impedances be denoted by Z_{i1} and Z_{i2}. Consider that the asymmetrical network is terminated with image impedance of port 2 Z_{i2} at its output pair of terminals then the impedance measured at its input pair of terminals will be image impedance of port 1 i.e. Z_{i1}. Similarly if port one is terminated in the image impedance of port 1 i.e. Z_{i1} then the impedance measured at port two will be the image impedance of port 2 i.e. Z_{i2}. These conditions are illustrated by the Fig. 8.5 (a) and (b).

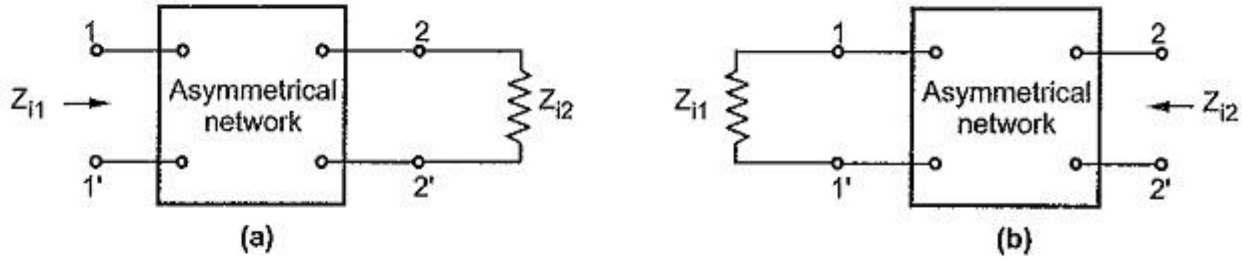


Fig. 8.5 Image impedances of asymmetrical network

When an asymmetrical network is terminated in image impedances at both the ports the network is called correctly terminated asymmetrical network as shown in the Fig. 8.6.

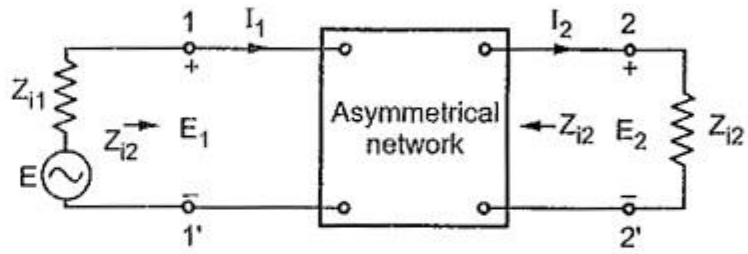


Fig. 8.6 Properly or correctly terminated asymmetrical network

Image transfer constant (e^{θ}):

When an asymmetrical network is terminated in its image impedances at both the ports as shown in the Fig. 8.6, then the ratio of currents I_1/I_2 will be different from the E_1/E_2.

Hence image transfer constant θ is defined as

$$e^{\theta} = \sqrt{\frac{E_1\, I_1}{E_2\, I_2}}$$

The real part of image transfer constant is called image attenuation constant; while the imaginary part is called image phase constant.